M14/5/MATHL/HP2/ENG/TZ1/XX/M

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MARKSCHEME

May 2014

MATHEMATICS

Higher Level

Paper 2

20 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to Scoris instructions and the document "**Mathematics HL: Guidance for e-marking May 2014**". It is essential that you read this document before you start marking. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by Scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 ***N* marks**

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 **Implied marks**

*Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 **Follow through marks**

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent** ***A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 **Mis-read**

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses one mark.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 **Discretionary marks *(d)***

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x)=2\sin(5x-3)$, the markscheme gives:

$f'(x)=\left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

Candidates should ***NO LONGER*** *be penalized for an accuracy error* ***(AP)***.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for ***FT***.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

SECTION A

1. **METHOD 1**

substituting
$-5+12\text{i}+a(2+3\text{i})+b=0$ *(A1)*
equating real or imaginary parts *(M1)*
$12+3a=0 \Rightarrow a=-4$ ***A1***
$-5+2a+b=0 \Rightarrow b=13$ ***A1***

METHOD 2

other root is $2-3\text{i}$ *(A1)*
considering either the sum or product of roots or multiplying factors *(M1)*
$4=-a$ (sum of roots) so $a=-4$ ***A1***
$13=b$ (product of roots) ***A1***

[4 marks]

2. $X : \text{N}\left(100, \sigma^2\right)$
$\text{P}(X<124)=0.68$ *(M1)(A1)*
$\frac{24}{\sigma}=0.4676....$ *(M1)*
$\sigma=51.315\ldots$ *(A1)*
$\text{variance}=2630$ ***A1***

[5 marks]

Notes: Accept use of $\text{P}(X<124.5)=0.68$ leading to variance $=2744$.

3. the number of ways of allocating presents to the first child is $\binom{7}{3}\left(\text{or }\binom{7}{2}\right)$ *(A1)*

multiplying by $\binom{4}{2}\left(\text{or }\binom{5}{3}\text{ or }\binom{5}{2}\right)$ *(M1)(A1)*

Note: Award ***M1*** for multiplication of combinations.

$\binom{7}{3}\binom{4}{2}=210$ ***A1***

[4 marks]

4. (a) $\begin{cases} x+2y-z=2 \\ 2x+y+z=1 \\ -x+4y+az=4 \end{cases}$

$\rightarrow \begin{cases} x+2y-z=2 \\ -3y+3z=-3 \\ 6y+(a-1)z=6 \end{cases}$ ***M1A1***

$\rightarrow \begin{cases} x+2y-z=2 \\ -3y+3z=-3 \\ (a+5)z=0 \end{cases}$ ***A1***

(or equivalent)

if not a unique solution then $a=-5$ ***A1***

> **Note:** The first ***M1*** is for attempting to eliminate a variable, the first ***A1*** for obtaining two expression in just two variables (plus *a*), and the second ***A1*** for obtaining an expression in just *a* and one other variable

[4 marks]

(b) if $a=-5$ there are an infinite number of solutions as last equation always true ***R1***

and if $a \neq -5$ there is a unique solution ***R1***

hence always a solution ***AG***

[2 marks]

Total [6 marks]

5. (a) $\frac{\pi}{2}(1.57), \frac{3\pi}{2}(4.71)$ ***A1A1***

hence the coordinates are $\left(\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$ ***A1***

[3 marks]

(b) (i) $\pi\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left(x^2 - (x + 2\cos x)^2\right)\mathrm{d}x$ ***A1A1A1***

Note: Award ***A1*** for $x^2 - (x + 2\cos x)^2$, ***A1*** for correct limits and ***A1*** for π.

(ii) $6\pi^2 (= 59.2)$ ***A2***

Notes: Do not award **ft** from (b)(i).

[5 marks]

Total [8 marks]

6. (a) **METHOD 1**

sketch showing where the lines cross or zeros of $y = x(x+2)^6 - x$ ***(M1)***
$x = 0$ ***(A1)***
$x = -1$ and $x = -3$ ***(A1)***
the solution is $-3 < x < -1$ or $x > 0$ ***A1A1***

Note: Do not award either final ***A1*** mark if strict inequalities are not given.

METHOD 2

separating into two cases $x > 0$ and $x < 0$ ***(M1)***
if $x > 0$ then $(x+2)^6 > 1 \Rightarrow$ always true ***(M1)***
if $x < 0$ then $(x+2)^6 < 1 \Rightarrow -3 < x < -1$ ***(M1)***
so the solution is $-3 < x < -1$ or $x > 0$ ***A1A1***

Note: Do not award either final ***A1*** mark if strict inequalities are not given.

METHOD 3

$f(x) = x^7 + 12x^6 + 60x^5 + 160x^4 + 240x^3 + 192x^2 + 64x$ ***(A1)***
solutions to $x^7 + 12x^6 + 60x^5 + 160x^4 + 240x^3 + 192x^2 + 63x = 0$ are ***(M1)***
$x = 0$, $x = -1$ and $x = -3$ ***(A1)***
so the solution is $-3 < x < -1$ or $x > 0$ ***A1A1***

Note: Do not award either final ***A1*** mark if strict inequalities are not given.

METHOD 4

$f(x) = x$ when $x(x+2)^6 = x$
either $x = 0$ or $(x+2)^6 = 1$ ***(A1)***
if $(x+2)^6 = 1$ then $x + 2 = \pm 1$ so $x = -1$ or $x = -3$ ***(M1)(A1)***
the solution is $-3 < x < -1$ or $x > 0$ ***A1A1***

Note: Do not award either final ***A1*** mark if strict inequalities are not given.

[5 marks]

continued ...

Question 6 continued

(b) **METHOD 1** (by substitution)

substituting $u = x + 2$ ***(M1)***

$\mathrm{d}u = \mathrm{d}x$

$\int (u-2)u^6 \mathrm{d}u$ ***M1A1***

$= \frac{1}{8}u^8 - \frac{2}{7}u^7 \ (+c)$ ***(A1)***

$= \frac{1}{8}(x+2)^8 - \frac{2}{7}(x+2)^7 \ (+c)$ ***A1***

METHOD 2 (by parts)

$u = x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1, \ \frac{\mathrm{d}v}{\mathrm{d}x} = (x+2)^6 \Rightarrow v = \frac{1}{7}(x+2)^7$ ***(M1)(A1)***

$\int x(x+2)^6 \,\mathrm{d}x = \frac{1}{7}x(x+2)^7 - \frac{1}{7}\int (x+2)^7 \,\mathrm{d}x$ ***M1***

$= \frac{1}{7}x(x+2)^7 - \frac{1}{56}(x+2)^8 \ (+c)$ ***A1A1***

METHOD 3 (by expansion)

$\int f(x)\,\mathrm{d}x = \int \left(x^7 + 12x^6 + 60x^5 + 160x^4 + 240x^3 + 192x^2 + 64x\right)\mathrm{d}x$ ***M1A1***

$= \frac{1}{8}x^8 + \frac{12}{7}x^7 + 10x^6 + 32x^5 + 60x^4 + 64x^3 + 32x^2 (+c)$ are ***M1A2***

Note: Award ***M1A1*** if at least four terms are correct.

[5 marks]

Total [10 marks]

7. if $n=0$

$7^3+2=345$ which is divisible by 5, hence true for $n=0$ ***A1***

> **Note:** Award ***A0*** for using $n=1$ but do not penalize further in question.

assume true for $n=k$ ***M1***

> **Note:** Only award the ***M1*** if truth is assumed.

so $7^{8k+3}+2=5p,\ p\in$ • ***A1***

if $n=k+1$

$7^{8(k+1)+3}+2$ ***M1***

$=7^8 7^{8k+3}+2$ ***M1***

$=7^8(5p-2)+2$ ***A1***

$=7^8.5p-2.7^8+2$

$=7^8.5p-11529600$

$=5\left(7^8p-2305920\right)$ ***A1***

hence if true for $n=k$, then also true for $n=k+1$. Since true for $n=0$, then true for all $n\in$ • ***R1***

[8 marks]

> **Note:** Only award the ***R1*** if the first two ***M1***s have been awarded.

8. (a) $\left(A\binom{6}{5}2^5 B+3\binom{6}{4}2^4 B^2\right)x^5$ ***M1A1A1***

$=\left(192AB+720B^2\right)x^5$ ***A1***

[4 marks]

(b) **METHOD 1**

$x=\frac{1}{6},\ A=\frac{3}{6}\left(=\frac{1}{2}\right),\ B=\frac{4}{6}\left(=\frac{2}{3}\right)$ ***A1A1A1***

probability is $\frac{4}{81}$ $(=0.0494)$ ***A1***

METHOD 2

$\mathrm{P}(5 \text{ eaten})=\mathrm{P}(\text{M eats 1})\ \mathrm{P}(\text{N eats 4})+\mathrm{P}(\text{M eats 0})\ \mathrm{P}(\text{N eats 5})$ ***(M1)***

$=\frac{1}{2}\binom{6}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2+\frac{1}{2}\binom{6}{5}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)$ ***(A1)(A1)***

$=\frac{4}{81}\ \left(=0.0494\right)$ ***A1***

[4 marks]

Total [8 marks]

9. (a) mean for week is 40.88 ***(A1)***

$P(S > 40) = 1 - P(S \leq 40) = 0.513$ ***A1***

[2 marks]

(b) $\dfrac{\text{probability there were more than 10 on Monday AND more than 40 over the week}}{\text{probability there were more than 10 on Monday}}$

M1

possibilities for the numerator are:

there were more than 40 birds on the power line on Monday ***R1***
11 on Monday and more than 29 over the course of the next 6 days ***R1***
12 on Monday and more than 28 over the course of the next 6 days … until
40 on Monday and more than 0 over the course of the next 6 days ***R1***
hence if *X* is the number on the power line on Monday and *Y*, the number on the power line Tuesday – Sunday then the numerator is ***M1***

$$P(X > 40) + P(X = 11) \times P(Y > 29) + P(X = 12) \times P(Y > 28) + \dots$$
$$+ P(X = 40) \times P(Y > 0)$$
$$= P(X > 40) + \sum_{r=11}^{40} P(X = r) P(Y > 40 - r)$$

hence solution is $\dfrac{P(X > 40) + \sum_{r=11}^{40} P(X = r) P(Y > 40 - r)}{P(X > 10)}$ ***AG***

[5 marks]

Total [7 marks]

SECTION B

10. (a) $x \to -\infty \Rightarrow y \to -\frac{1}{2}$ so $y = -\frac{1}{2}$ is an asymptote ***(M1)A1***

$e^x - 2 = 0 \Rightarrow x = \ln 2$ so $x = \ln 2 (= 0.693)$ is an asymptote ***(M1)A1***

[4 marks]

(b) (i) $f'(x) = \frac{2(e^x - 2)e^{2x} - (e^{2x} + 1)e^x}{(e^x - 2)^2}$ ***M1A1***

$= \frac{e^{3x} - 4e^{2x} - e^x}{(e^x - 2)^2}$

(ii) $f'(x) = 0$ when $e^{3x} - 4e^{2x} - e^x = 0$ ***M1***

$e^x(e^{2x} - 4e^x - 1) = 0$

$e^x = 0,\ e^x = -0.236,\ e^x = 4.24\ \left(\text{or } e^x = 2 \pm \sqrt{5}\right)$ ***A1A1***

> **Note:** Award ***A1*** for zero, ***A1*** for other two solutions.
> Accept any answers which show a zero, a negative and a positive.

as $e^x > 0$ exactly one solution ***R1***

> **Note:** Do not award marks for purely graphical solution.

(iii) $(1.44, 8.47)$ ***A1A1***

[8 marks]

(c) $f'(0) = -4$ ***(A1)***

so gradient of normal is $\frac{1}{4}$ ***(M1)***

$f(0) = -2$ ***(A1)***

so equation of L_1 is $y = \frac{1}{4}x - 2$ ***A1***

[4 marks]

continued ...

Question 10 continued

(d) $f'(x) = \frac{1}{4}$ ***M1***

so $x = 1.46$ ***(M1)A1***

$f(1.46) = 8.47$ ***(A1)***

equation of L_2 is $y - 8.47 = \frac{1}{4}(x - 1.46)$ ***A1***

(or $y = \frac{1}{4}x + 8.11$)

[5 marks]

Total [21 marks]

11. (a) $\int_2^3 (ax+b)\,dx\,(=1)$ ***M1A1***

$\left[\frac{1}{2}ax^2+bx\right]_2^3 (=1)$ ***A1***

$\frac{5}{2}a+b=1$ ***M1***

$5a+2b=2$ ***AG***

[4 marks]

(b) (i) $\int_2^3 \left(ax^2+bx\right)dx\,(=\mu)$ ***M1A1***

$\left[\frac{1}{3}ax^3+\frac{1}{2}bx^2\right]_2^3 (=\mu)$ ***A1***

$\frac{19}{3}a+\frac{5}{2}b=\mu$ ***A1***

eliminating b ***M1***

eg

$\frac{19}{3}a+\frac{5}{2}\left(1-\frac{5}{2}a\right)=\mu$ ***A1***

$\frac{1}{12}a+\frac{5}{2}=\mu$

$a=12\mu-30$ ***AG***

Note: Elimination of *b* could be at different stages.

(ii) $b=1-\frac{5}{2}(12\mu-30)$

$=76-30\mu$ ***A1***

Note: This solution may be seen in part (i).

[7 marks]

(c) (i) $\int_2^{2.3} (ax+b)\,dx\,(=0.5)$ ***(M1)(A1)***

$\left[\frac{1}{2}ax^2+bx\right]_2^{2.3} (=0.5)$

$0.645a+0.3b\,(=0.5)$ ***(A1)***

$0.645(12\mu-30)+0.3(76-30\mu)=0.5$ ***M1***

$\mu=2.34\left(=\frac{295}{126}\right)$ ***A1***

continued …

Question 11 continued

(ii) $\mathrm{E}\left(X^2\right)=\int_2^3 x^2\,(ax+b)\,\mathrm{d}x$ ***(M1)***

$a=12\mu-30=-\dfrac{40}{21},\ b=76-30\mu=\dfrac{121}{21}$ ***(A1)***

$\mathrm{E}\left(X^2\right)=\int_2^3 x^2\left(-\dfrac{40}{21}x+\dfrac{121}{21}\right)\mathrm{d}x=5.539\ldots\left(=\dfrac{349}{63}\right)$ ***(A1)***

$\mathrm{Var}\,(X)=5.539\mathrm{K}\ -(2.341\mathrm{K}\ )^2=0.05813\ldots$ ***(M1)***

$\sigma=0.241$ ***A1***

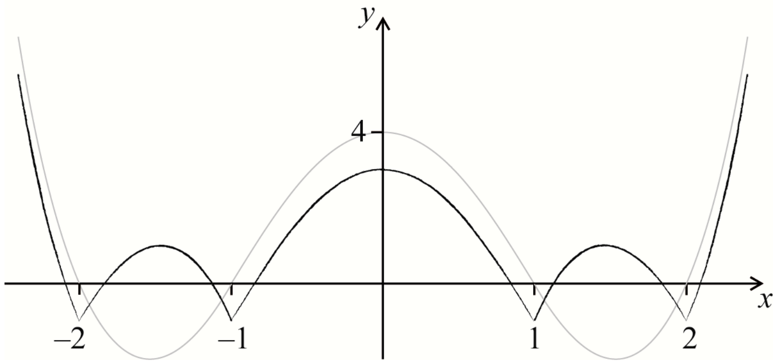
[10 marks]

Total [21 marks]

12. (a) (i) $f(0) = -1$ ***(M1)A1***

(ii) $(f \circ g)(0) = f(4) = 3$ ***A1***

(iii)

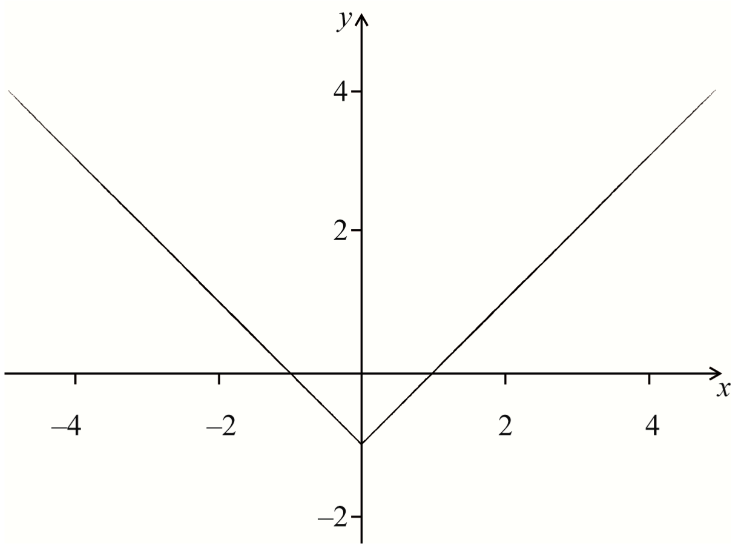


(M1)A1

Note:	Award ***M1*** for evidence that the lower part of the graph has been reflected and ***A1*** correct shape with *y*-intercept below 4.

[5 marks]

(b) (i)



(M1)A1

Note:	Award ***M1*** for any translation of $y = \lvert x \rvert$.

(ii) ± 1 ***A1***

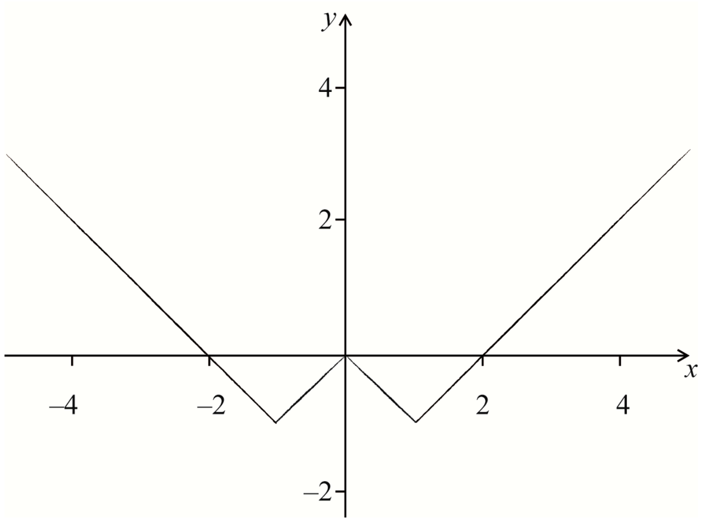
Note:	Do not award the ***A1*** if coordinates given, but do not penalise in the rest of the question

[3 marks]

continued ...

Question 12 continued

(c) (i)



(M1)A1

Note: Award ***M1*** for evidence that lower part of (b) has been reflected in the *x*-axis and translated.

(ii) $0, \pm 2$ ***A1***

[3 marks]

(d) (i) $\pm 1, \pm 3$ ***A1***

(ii) $0, \pm 2, \pm 4$ ***A1***

(iii) $0, \pm 2, \pm 4, \pm 6, \pm 8$ ***A1***

[3 marks]

(e) (i) $(1, 3), (2, 5), \ldots$ ***(M1)***

$N = 2n + 1$ ***A1***

(ii) Using the formula of the sum of an arithmetic series ***(M1)***

EITHER

$$4(1+2+3+\ldots+n) = \frac{4}{2}n(n+1)$$

$= 2n(n+1)$ ***A1***

OR

$$2(2+4+6+\ldots+2n) = \frac{2}{2}n(2n+2)$$

$= 2n(n+1)$ ***A1***

[4 marks]

Total [18 marks]